The "Cloudlet" Effective Medium Model for Sub-Grid Scale Random 3D Inhomogeneities

Grant Petty
Atmospheric and Oceanic Sciences
U. Wisconsin-Madison

- Reference:
  - G.W. Petty, 2002: Area-average solar radiative transfer in three-dimensionally inhomogeneous clouds: the Independently Scattering Cloudlet model. *JAS*, 59, 2910-2929
- Related (but independent) literature:
  - Cairns et al. (2000) - rescaling
  - Hobson and Padman (1993) – “mega-grains” approximation for interstellar dust

Motivation

- Simple **conceptual model** of how sub-grid scale 3-D inhomogeneities influence radiative transfer through a cloudy volume.
- Computational framework for **parameterizing** the effects of "lumpiness" in a scattering medium

Inspiration

- Countless hours of staring at chaotic clouds out of airplane windows: Is there a **simple unifying principle** to describe the area- or volume-averaged radiative transfer?
- Thought experiment: A **tractable** (though patently unrealistic) version of the random 3-D problem that can be “solved.”
- Generalization: Does the tractable version have any qualitative or even quantitative **applicability to real cloud structures?**

What this method/model is *not*

- *Not* a explicit 3-D radiative transfer code
- *Not* a ready-to-use parameterization scheme for GCMs (but may have relevance to such schemes)
- *Not* a framework for addressing
  - multilayer clouds
  - 2-D inhomogeneity (apart from special case)
  - gaseous absorption
- *Complements*, rather than replacing, IPA

What this method/model *is*

A physically self-consistent framework for

- *jointly* scaling the extinction and scattering properties of a cloud volume to account for *sub-grid scale* inhomogeneities, and
- predicting the *functional dependence* of the above properties on variable absorption (for fixed cloud structure).

Key predictions and empirical results

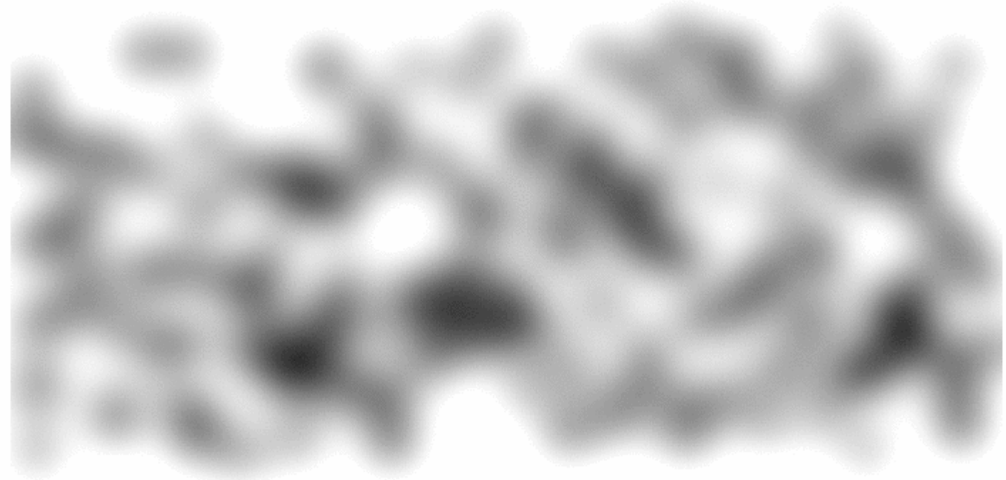
- Geometric scales don’t matter – only optical scales
- Cloud water variance is not a relevant property for RT purposes..
- Inhomogeneities with optical dimensions << 1 don’t matter
- Simple geometric models can sometimes be excellent proxies for complex/chaotic structures.

Development

- Derivation for highly idealized (unrealistic) geometric structures
- Empirical tests of applicability to non-ideal cloud structures
- Investigation of mapping between abstract model parameters and measurable cloud properties. (barely begun)

Realistic

a)



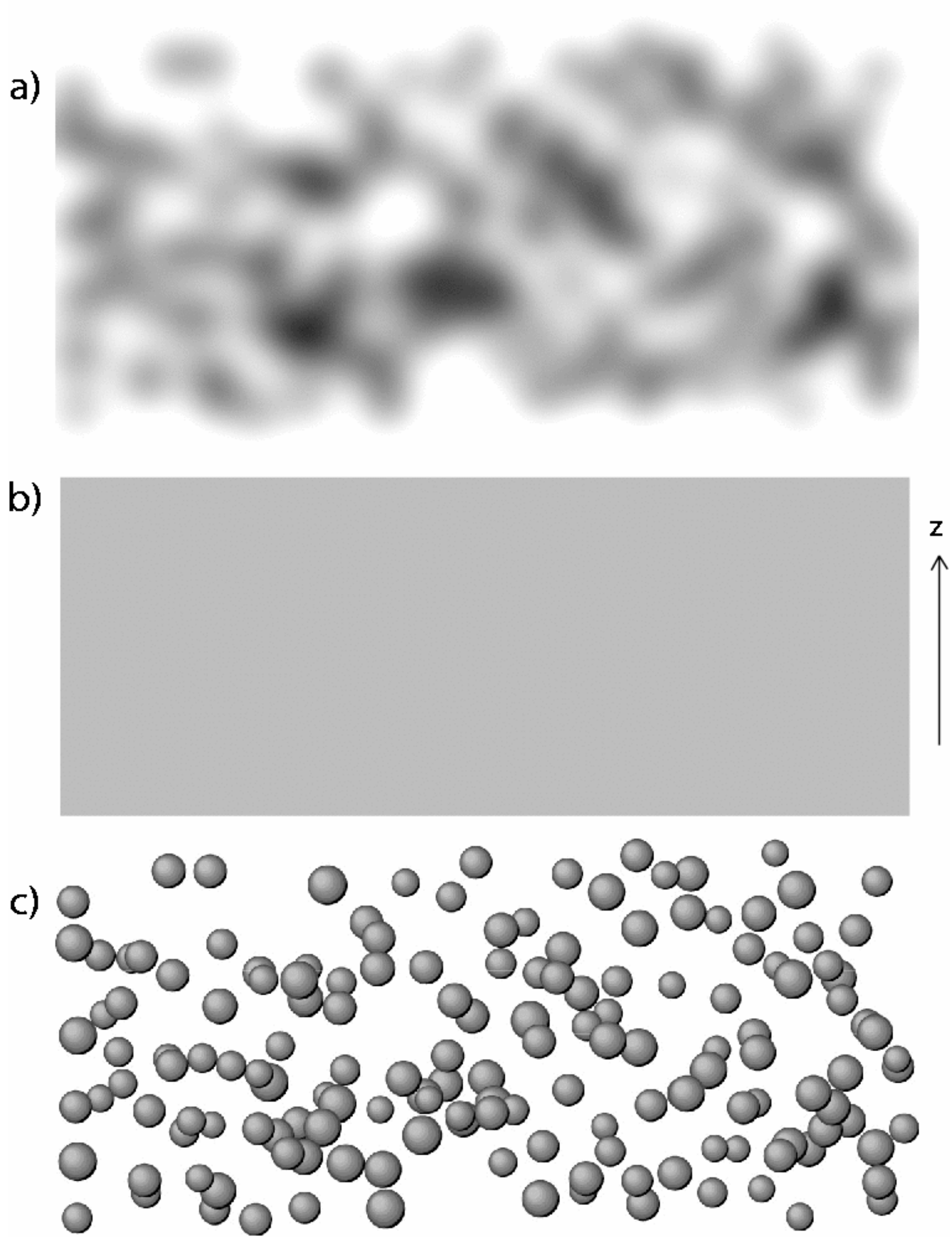
Realistic

Homogeneous
(plane parallel)

Realistic

Homogeneous
(plane parallel)

Independently
scattering cloudlets
(ISC) model



Even "homogeneous clouds aren't really homogeneous – they consist of distributions of individually scattering (and absorbing) cloud droplets.

"Bulk" radiative properties are traditionally computed as follows:

$$k = \int_0^\infty C_{\mathrm{ext}}(r) n(r)\, dr, \tag{1}$$

$$\varpi_0 = \frac{1}{k} \int_0^\infty C_{\mathrm{sca}}(r) n(r)\, dr, \tag{2}$$

$$P(\Theta) = \frac{1}{\varpi_0 k} \int_0^\infty C_{\mathrm{sca}}(r) P(\Theta;\, r) n(r)\, dr, \tag{3}$$

$$w = \rho_l \int_0^\infty \frac{4}{3} \pi r^3 n(r)\, dr, \tag{6}$$

With the Independently Scattering Cloudlets (ISC) model, we adapt the same approach to 3-D collections of *macroscopic* cloud elements.

Goal is to find a physically self-consistent way to adjust the optical depth, single scatter albedo, AND scattering asymmetry parameter to account for inhomogeneities within a cloud volume.

Simple example: Beer's law defines relationship between plane parallel optical depth and direct transmittance.

$$T_{\mathrm{dir}} = \exp\left[-\frac{\tau^*}{\mu_0}\right], \tag{9}$$

If we consider the AREA-AVERAGE direct transmittance for an inhomogeneous cloud layer, we can invert Beer's Law to find an "effective" optical depth.

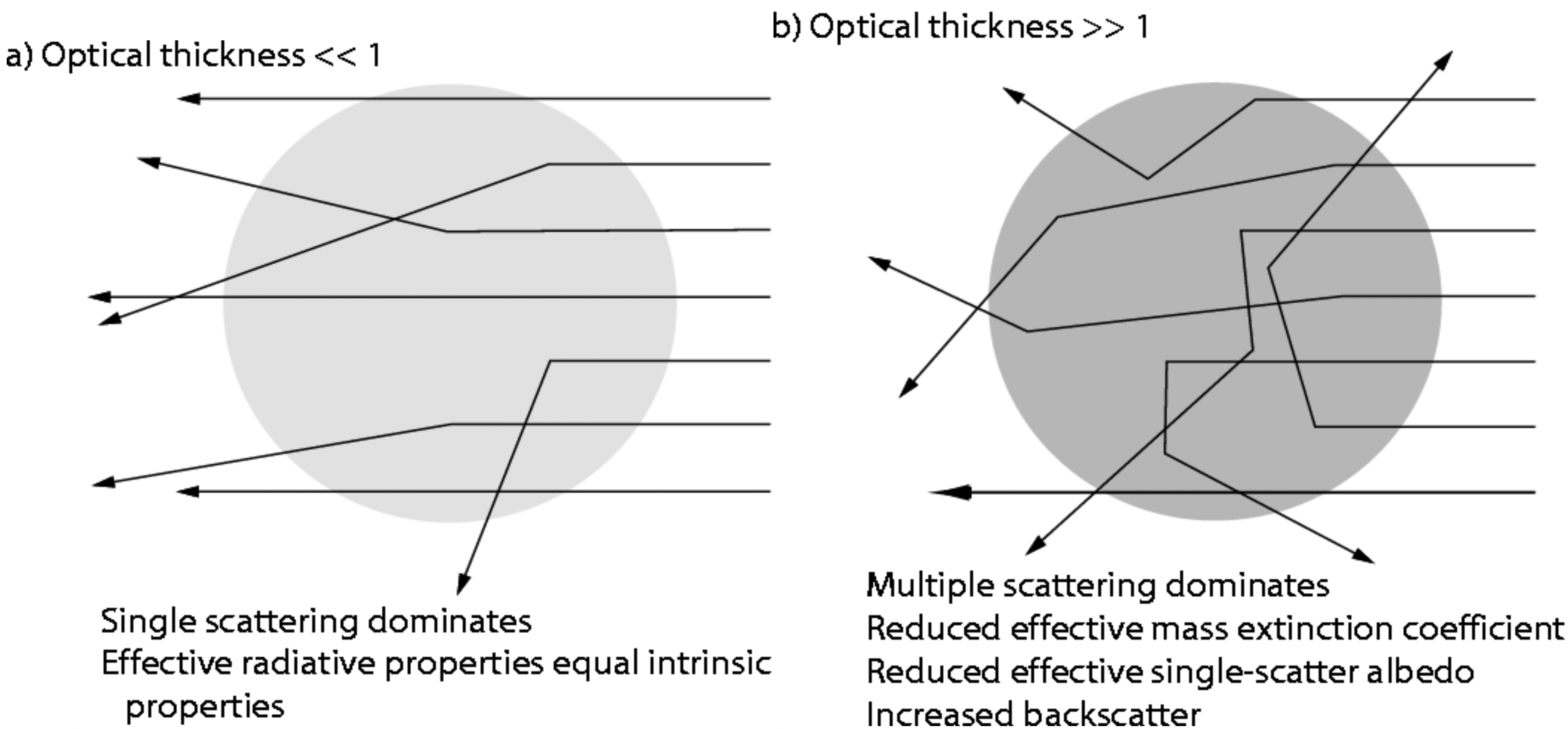
In general, the adjusted optical depth will be smaller than the the "true" area-averaged optical depth.

More generally,

- •Reduce optical depth
- •Reduce single scatter albedo
- •Reduce scattering asymmetry

Adjustments must be made in a physically self-consistent way!

ISC model provides a basis for making those adjustments


a) Optical thickness << 1
b) Optical thickness >> 1
Single scattering dominates
Effective radiative properties equal intrinsic properties
Multiple scattering dominates
Reduced effective mass extinction coefficient
Reduced effective single-scatter albedo
Increased backscatter

Need to determine extinction cross-sections etc. for individual "cloudlets" and sum over volume to get bulk radiative transfer properties

$$M_i = \frac{4}{3}\pi R_i^3 w_i. \tag{11}$$

$$\overline{w} = \frac{1}{V}\sum_i M_i \tag{12}$$

$$k'' = \frac{1}{V}\sum_i C'_{\mathrm{ext},i} \tag{13}$$

$$\sigma'' = \frac{k''}{\overline{w}} \tag{14}$$

$$\varpi_0'' = \frac{1}{Vk''}\sum_i C'_{\mathrm{ext},i}\varpi'_{0_i} \tag{15}$$

$$P''(\Theta) = \frac{1}{Vk''\varpi_0}\sum_i C'_{\mathrm{ext},i}\varpi'_{0_i}P'_i(\Theta) \tag{16}$$

$$g'' = \frac{1}{Vk''\varpi_0}\sum_i C'_{\mathrm{ext},i}\varpi'_{0_i}g'_i. \tag{17}$$

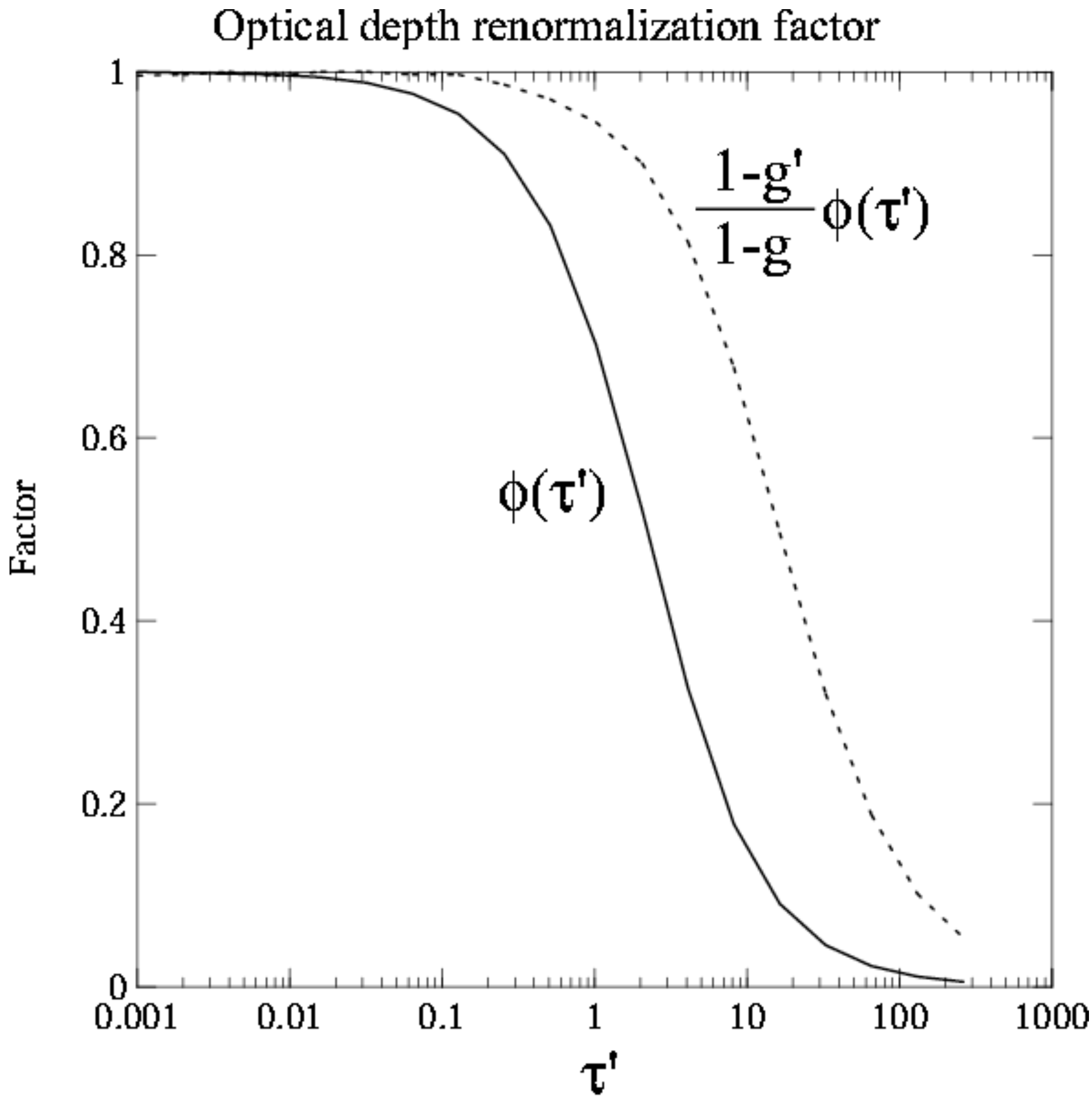
Extinction cross-section of cloudlet is easily dealt with, if you assume a homogeneous spherical cloudlet with arbitrary "intrinsic" scattering and extinction properties:

$$\tau' \equiv 2kR = 2\sigma wR = \frac{3\sigma M}{2\pi R^2}. \qquad (18)$$

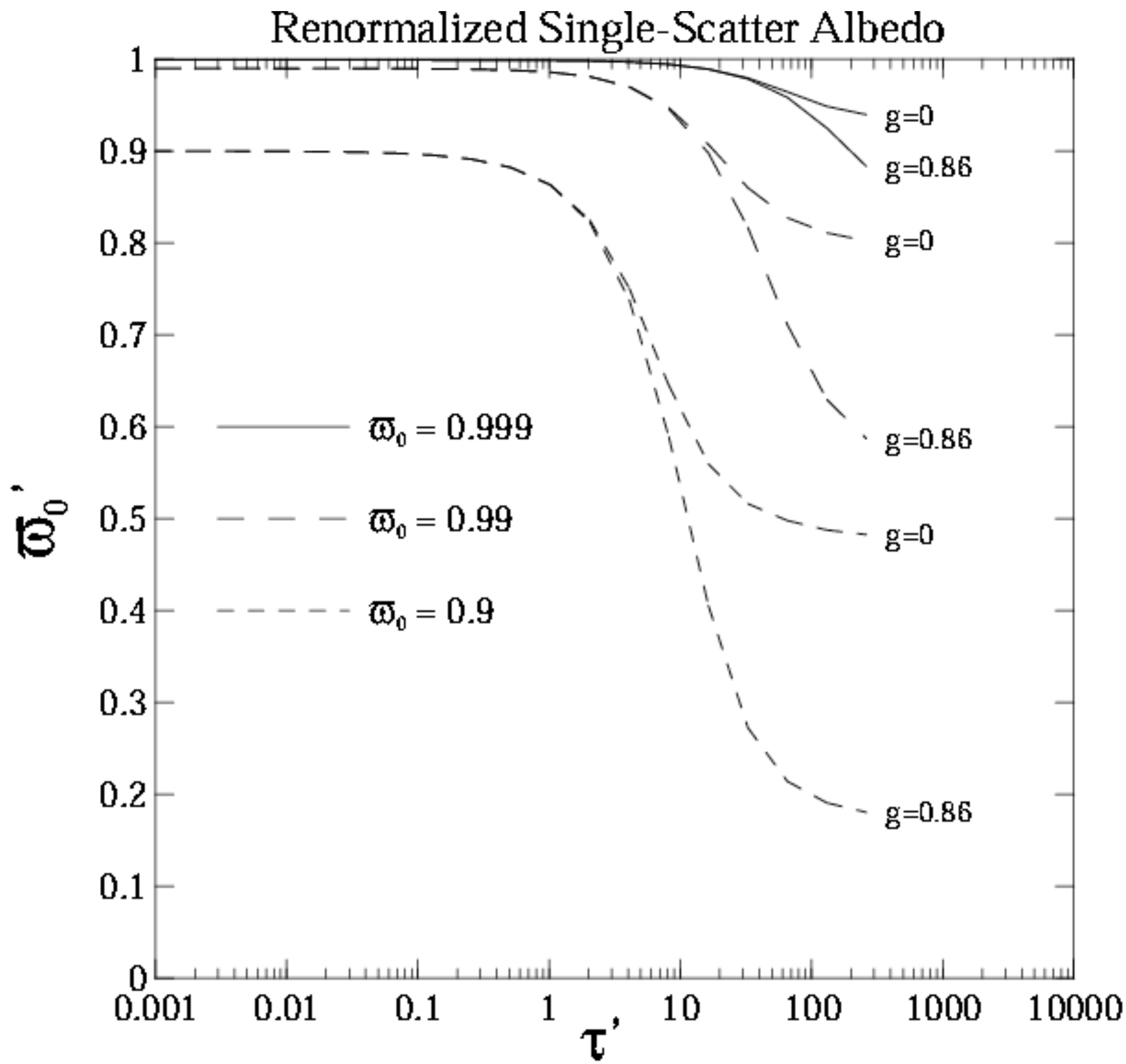
$$C'_{\text{ext}} = \int_0^R 2\pi r\{1 - \exp[-2k(R^2 - r^2)^{1/2}]\}\, dr, \qquad (19)$$

$$C'_{\text{ext}} = \frac{\pi}{2k^2}[2k^2R^2 + (2kR + 1)e^{-2kR} - 1]. \qquad (20)$$

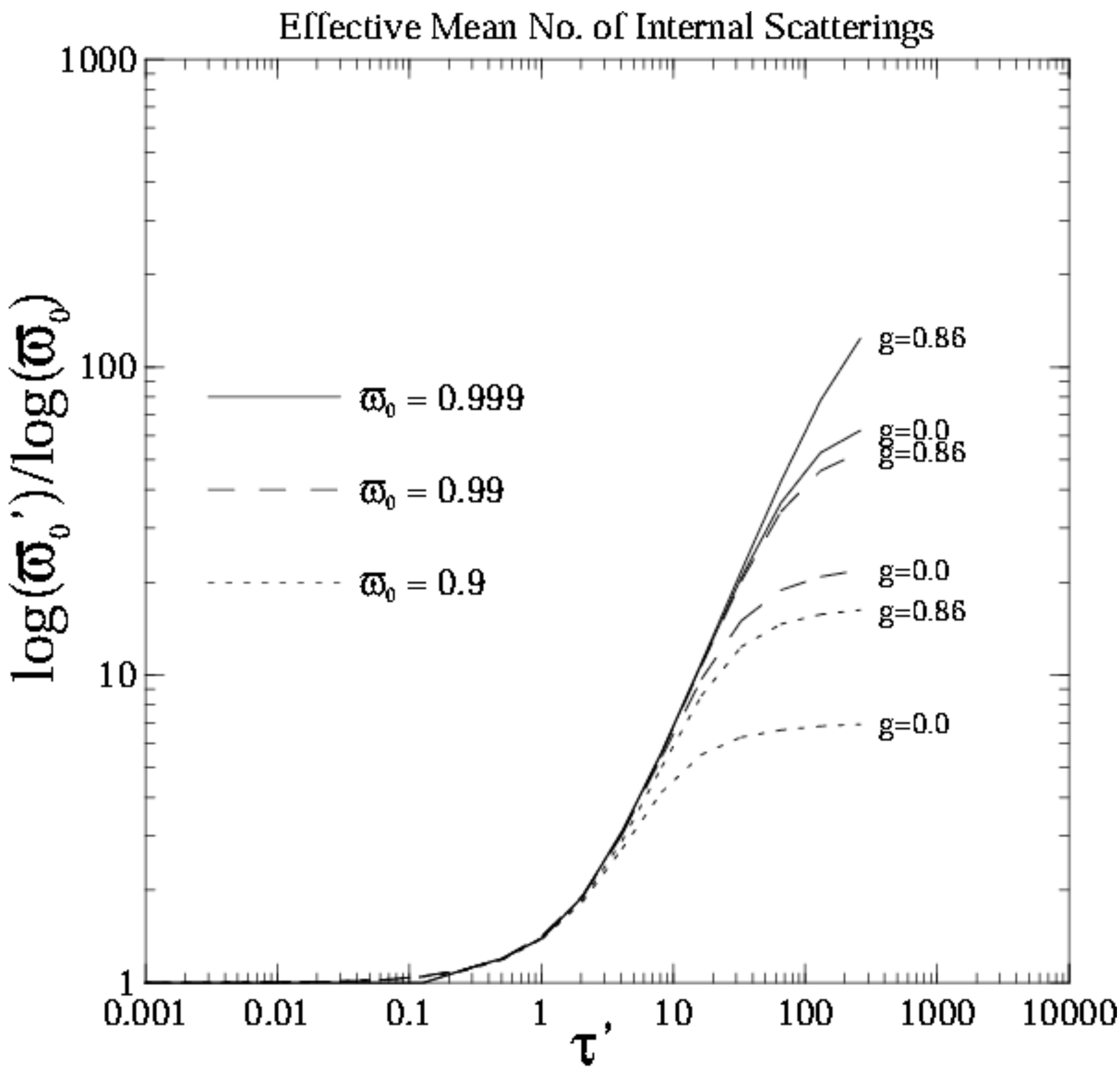
$$\boxed{\phi(\tau') = \frac{6[(\tau' + 1)e^{-\tau'} - 1] + 3\tau'^2}{2\tau'^3}. \qquad (22)}$$



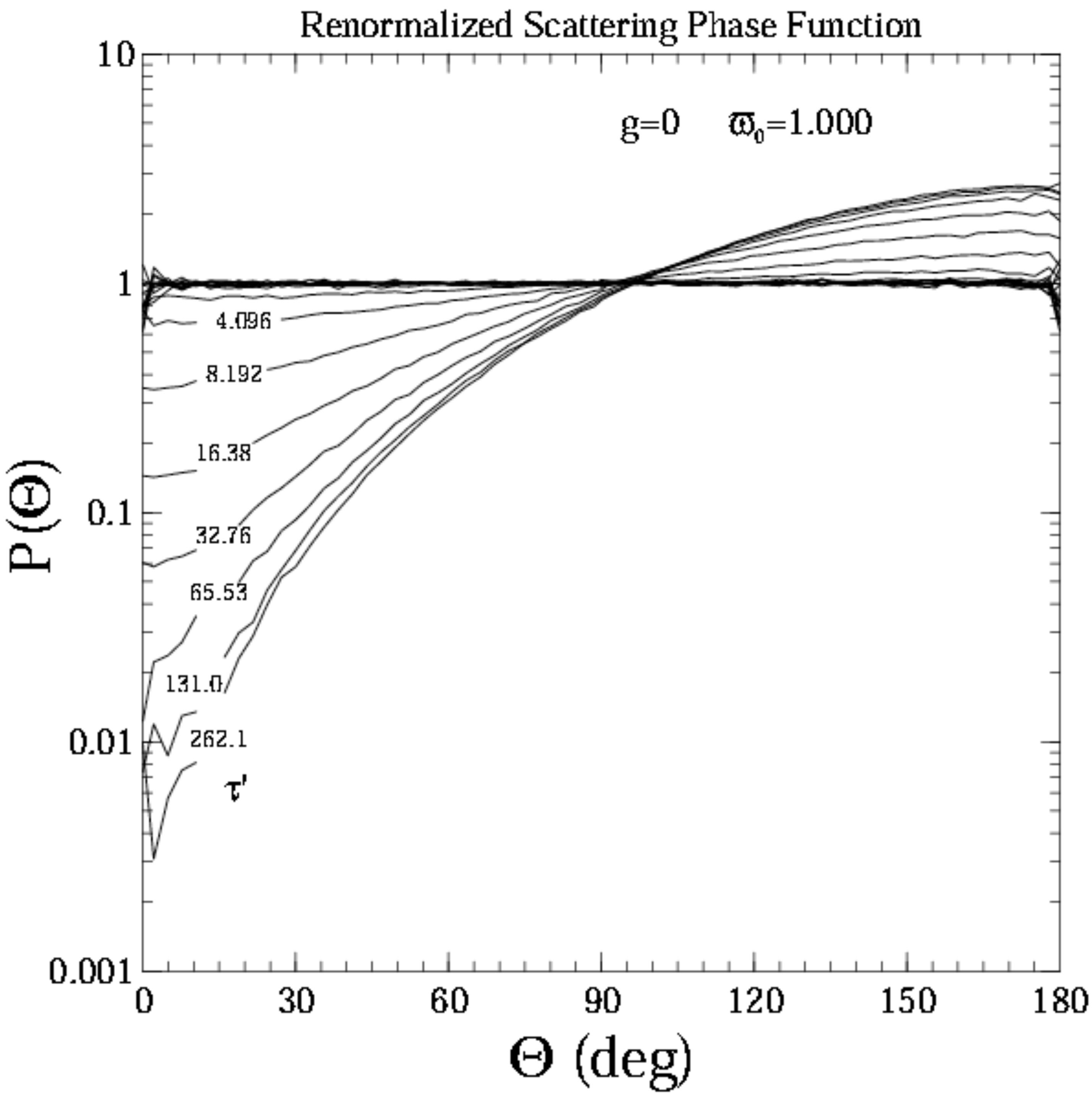
Optical depth renormalization factor
Factor
τ'
0
0.2
0.4
0.6
0.8
1
0.001
0.01
0.1
1
10
100
1000
$\phi(\tau')$
$\frac{1-g'}{1-g}\phi(\tau')$



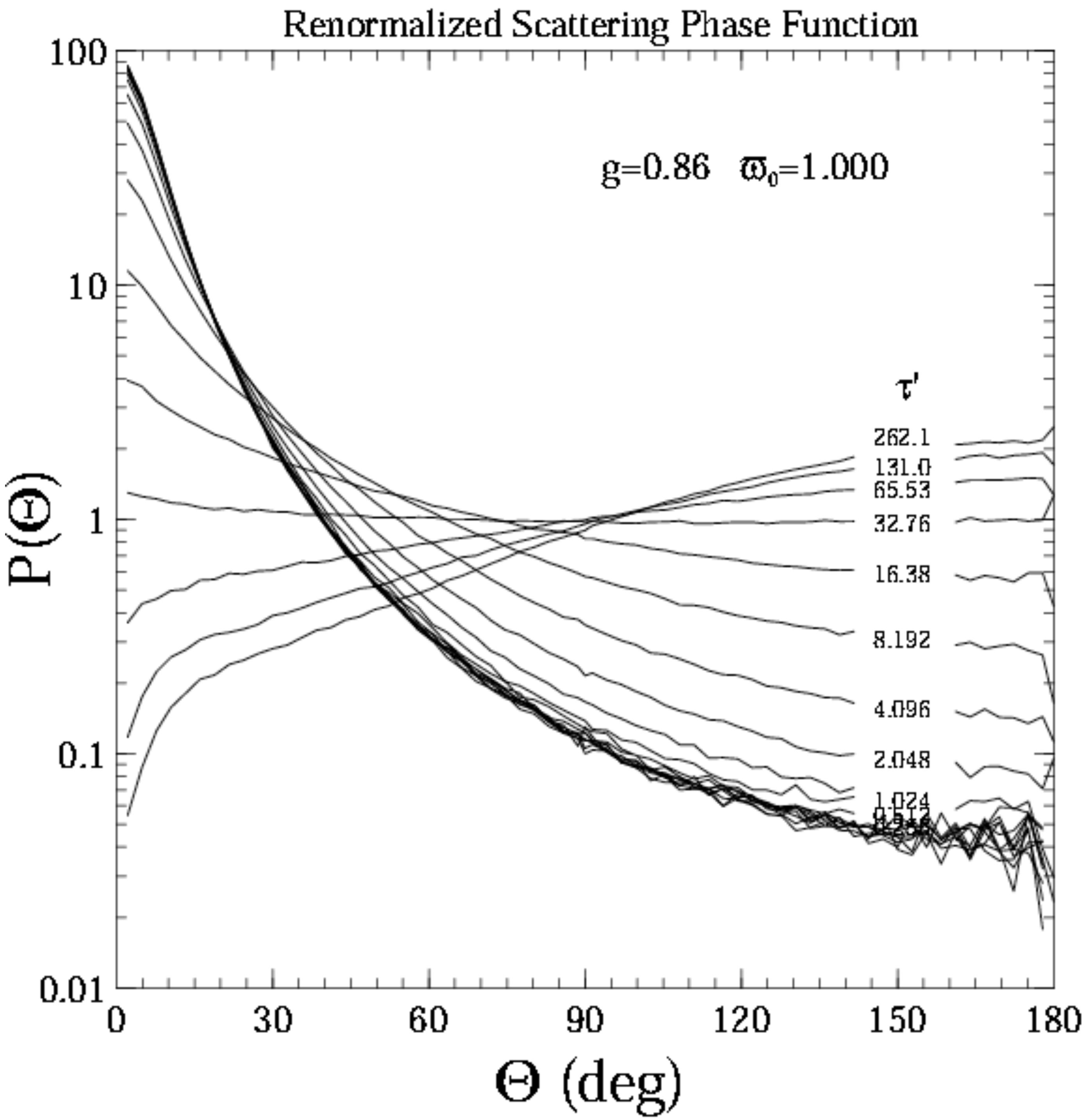
Renormalized Single-Scatter Albedo
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0.001
0.01
0.1
1
10
100
1000
10000
ϖ0'
τ'
ϖ0 = 0.999
ϖ0 = 0.99
ϖ0 = 0.9
g=0
g=0.86
g=0
g=0.86
g=0
g=0.86



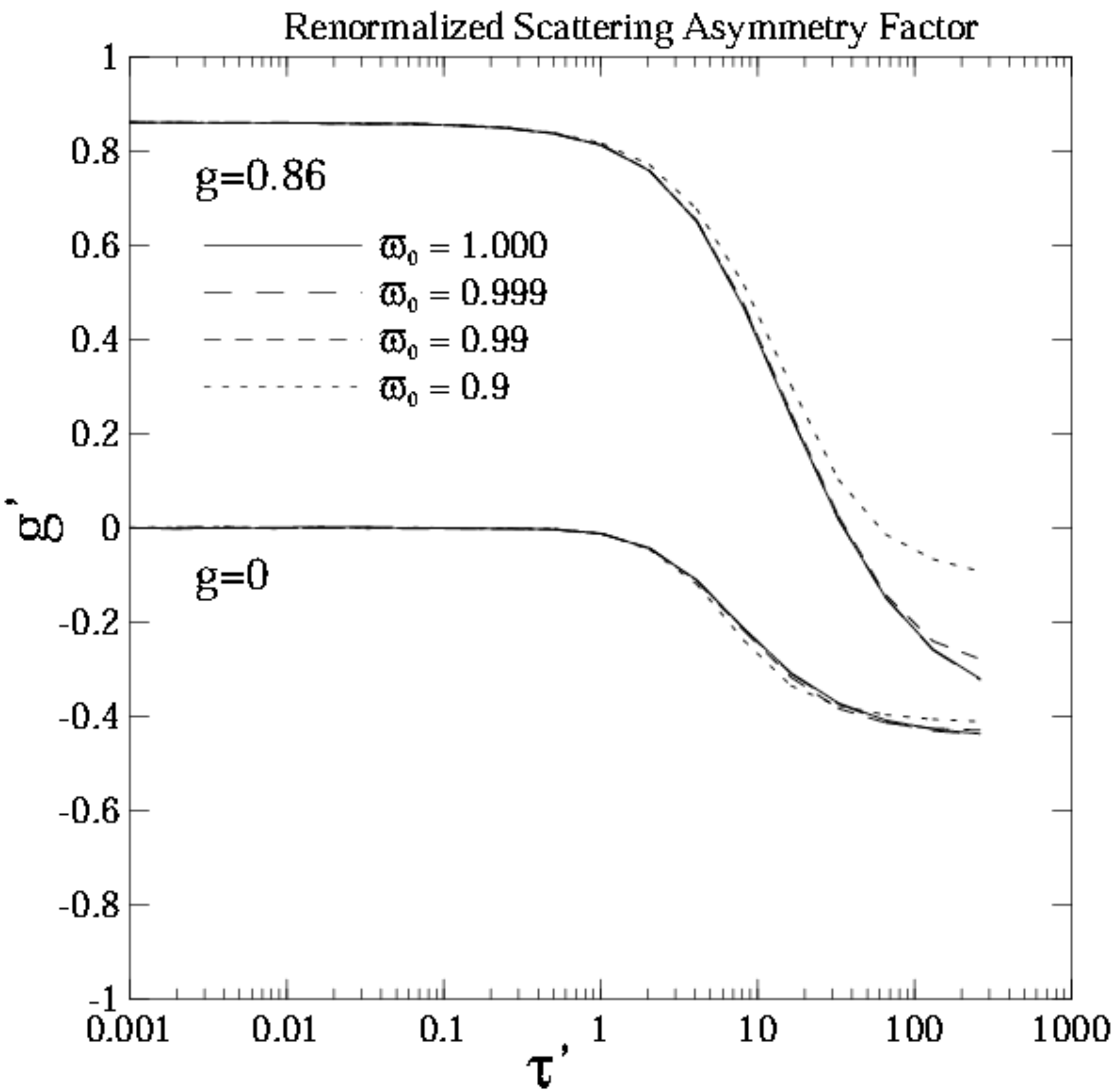
Effective Mean No. of Internal Scatterings
1000
100
10
1
0.001
0.01
0.1
1
10
100
1000
10000
τ'
log(ϖ₀')/log(ϖ₀)
ϖ₀ = 0.999
ϖ₀ = 0.99
ϖ₀ = 0.9
g=0.86
g=0.0
g=0.86
g=0.0
g=0.86
g=0.0



Renormalized Scattering Phase Function
g=0
ϖ0=1.000
4.096
8.192
16.38
32.76
65.53
131.0
262.1
τ'
P(Θ)
Θ (deg)
10
1
0.1
0.01
0.001
0
30
60
90
120
150
180



Renormalized Scattering Phase Function
g=0.86 ϖ_0=1.000
τ'
262.1
131.0
65.53
32.76
16.38
8.192
4.096
2.048
1.024
100
10
1
0.1
0.01
0
30
60
90
120
150
180
P(Θ)
Θ (deg)



Renormalized Scattering Asymmetry Factor
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
g'
g=0.86
g=0
ϖ_0 = 1.000
ϖ_0 = 0.999
ϖ_0 = 0.99
ϖ_0 = 0.9
0.001
0.01
0.1
1
10
100
1000
τ'

Validation of fundamental concept

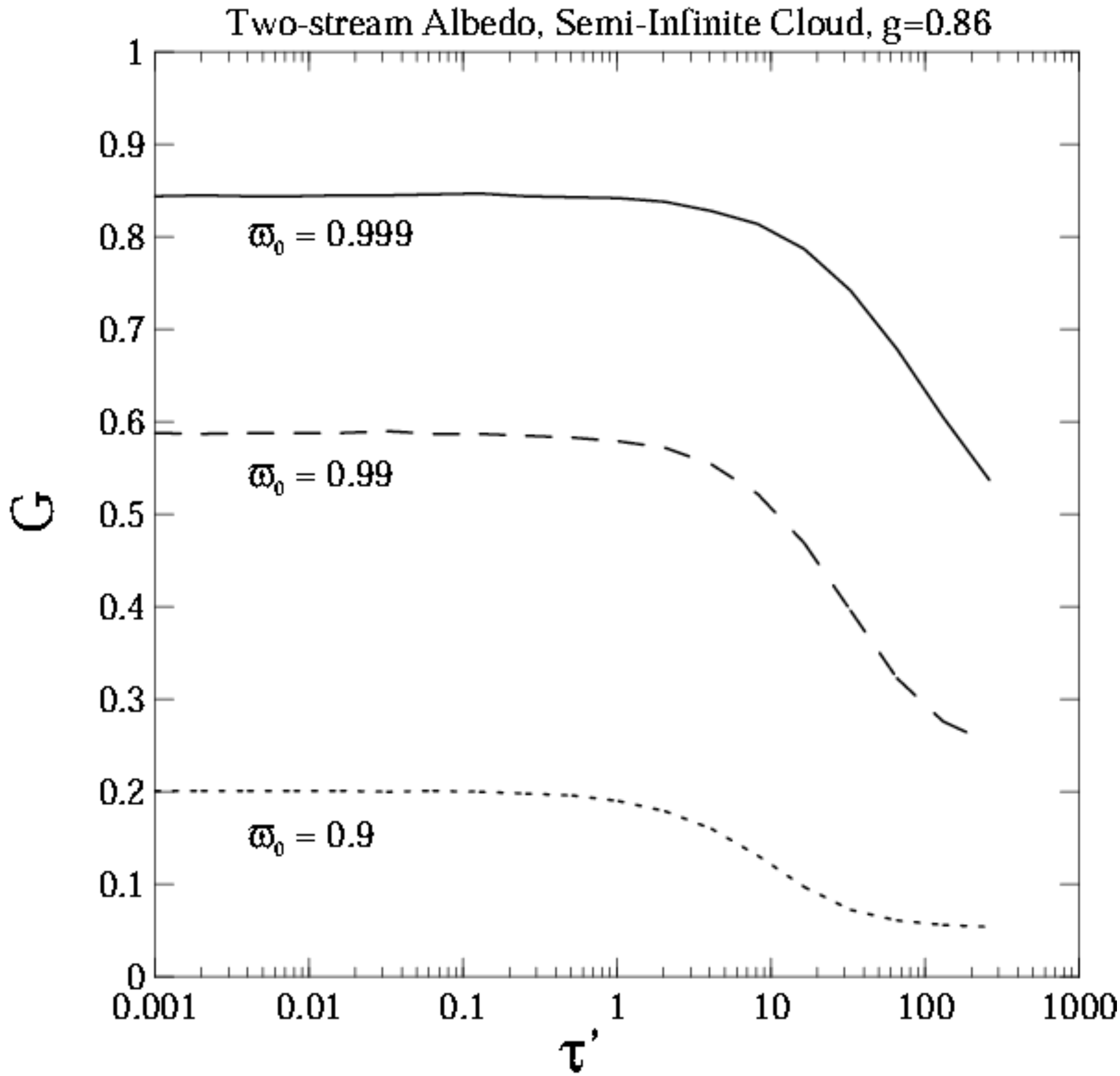
Compare ISC predictions with explicit Monte Carlo calculation

- Monte Carlo
  - Randomly populate 3-D domain with cubes of optical diameter $\tau' = 8$
  - Cloudy:clear ratio is 1:8
  - Area average optical depth = 16
  - Periodic extension to horizontally infinite domain
- ISC
  - Use corresponding ISC-derived scaled properties in DISORT to compute fluxes

Results

(see Table 1 for details)

- Monte Carlo results for test case are essentially identical to ISC results!
- Plane parallel results for same area-averaged optical depth are seriously in error.



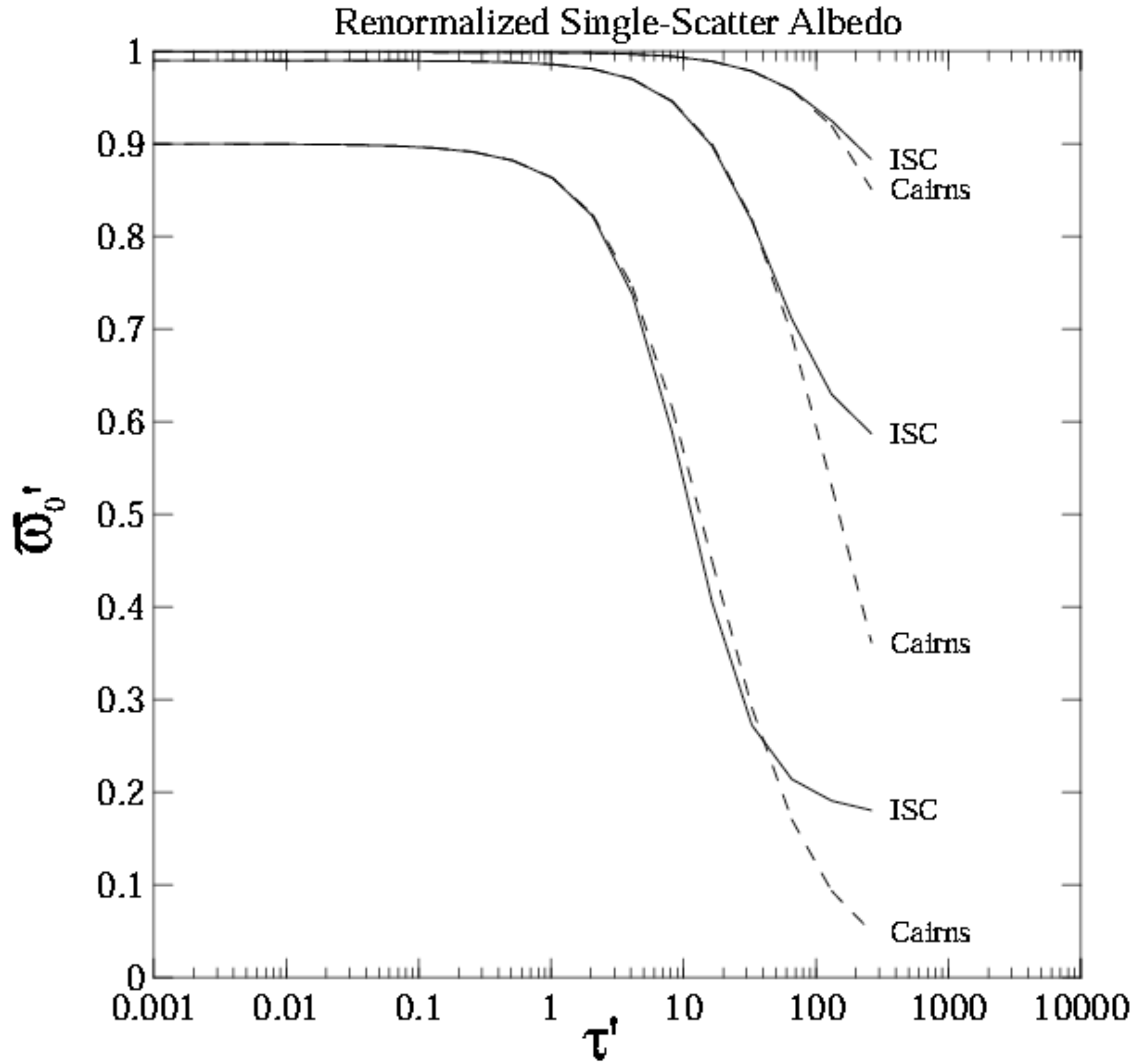
Two-stream Albedo, Semi-Infinite Cloud, g=0.86
G
τ'
ϖ0 = 0.999
ϖ0 = 0.99
ϖ0 = 0.9
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0.001
0.01
0.1
1
10
100
1000

Cairns et al (2000) renormalization: same objective, vastly different method (perturbative expansion of RTE, assuming log-normal cloud water density and other restrictive assumptions). Key parameter is $V = 1 - \exp(\delta^2)$

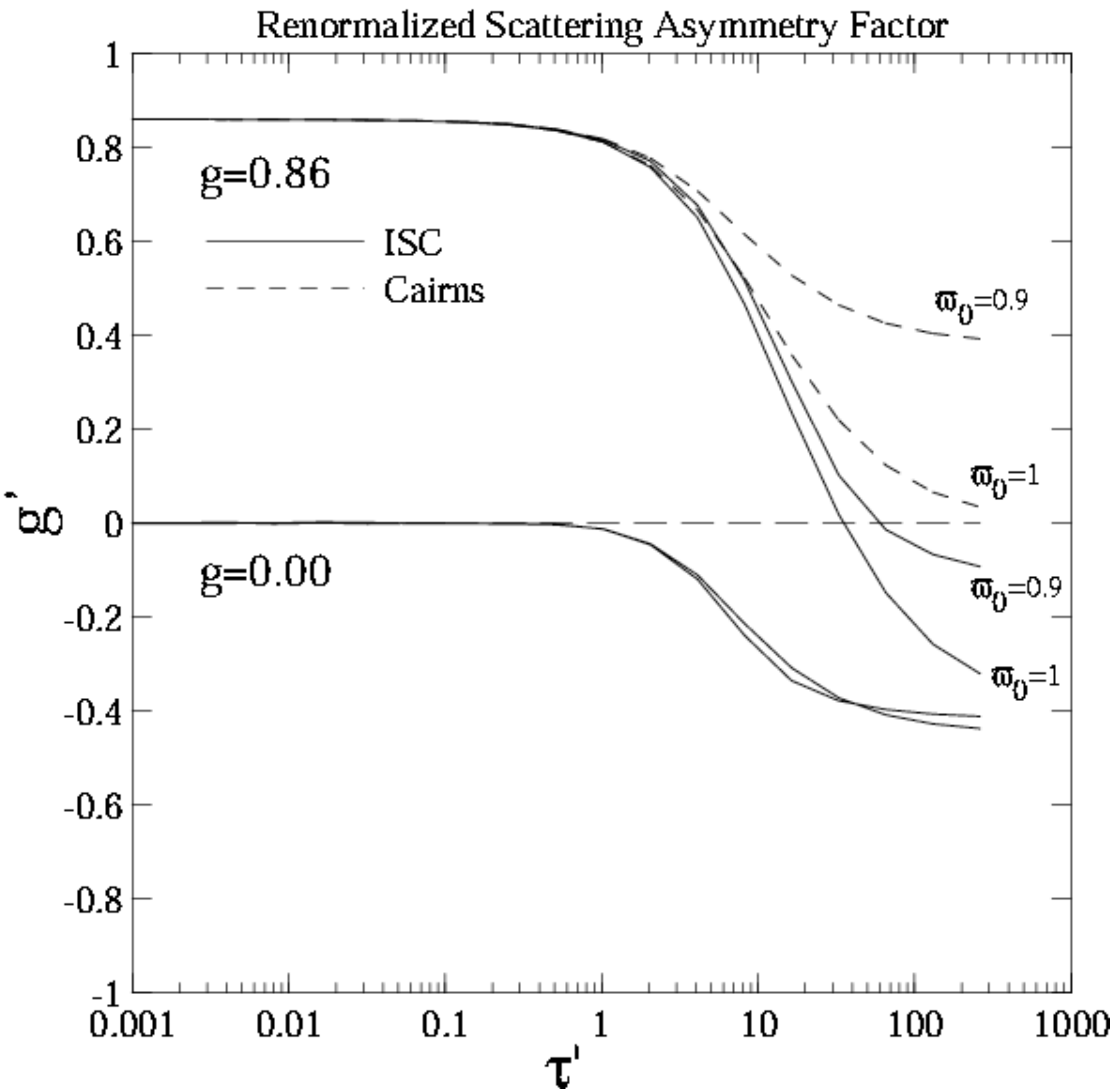
$$\sigma' = \sigma(1 - V)^{-1}, \tag{32}$$

$$\varpi_0' = \varpi_0/[1 + V(1 - \varpi_0)], \tag{33}$$

$$g' = g[1 + V(1 - \varpi_0)]/[1 + V(1 - \varpi_0 g)], \tag{34}$$



Renormalized Single-Scatter Albedo
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0.001
0.01
0.1
1
10
100
1000
10000
ϖ_0'
τ'
ISC
Cairns
ISC
Cairns
ISC
Cairns



Renormalized Scattering Asymmetry Factor
g=0.86
ISC
Cairns
g=0.00
ϖ0=0.9
ϖ0=1
ϖ0=0.9
ϖ0=1
g'
τ'
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0.001
0.01
0.1
1
10
100
1000

Two-parameter generalization of ISC model:

f defines fraction of inhomogeneous cloud water

tau' defines optical diameter of inhomogeneous elements

$$W_1 = f\overline{W}, \tag{35}$$

$$W_2 = (1 - F)\overline{W}. \tag{36}$$

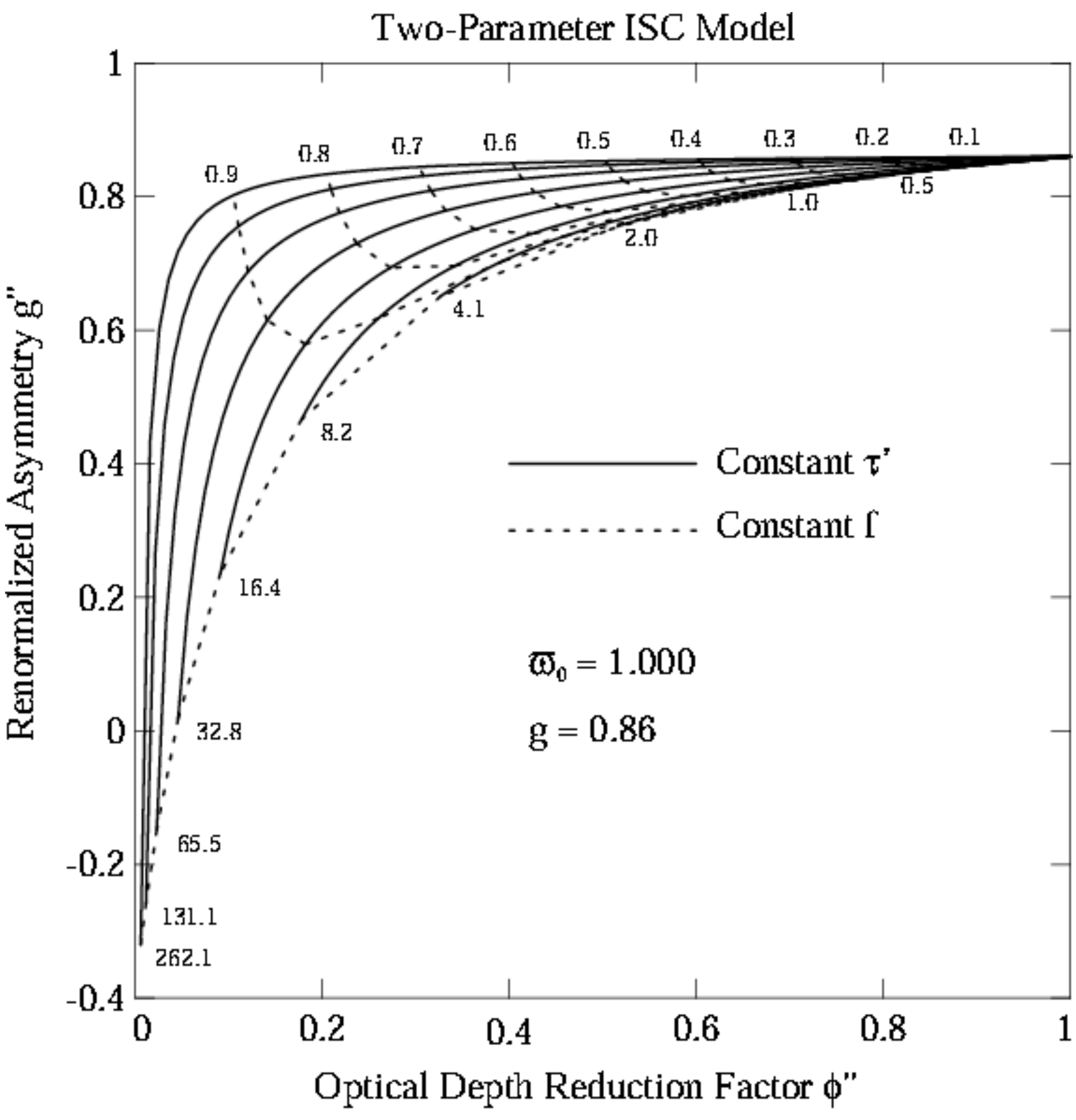
$$\tau_{\text{hom}} = (1 - f)\sigma\overline{W} \tag{40}$$

$$\tau_{\text{inhom}} = f\phi(\tau')\sigma\overline{W}, \tag{41}$$

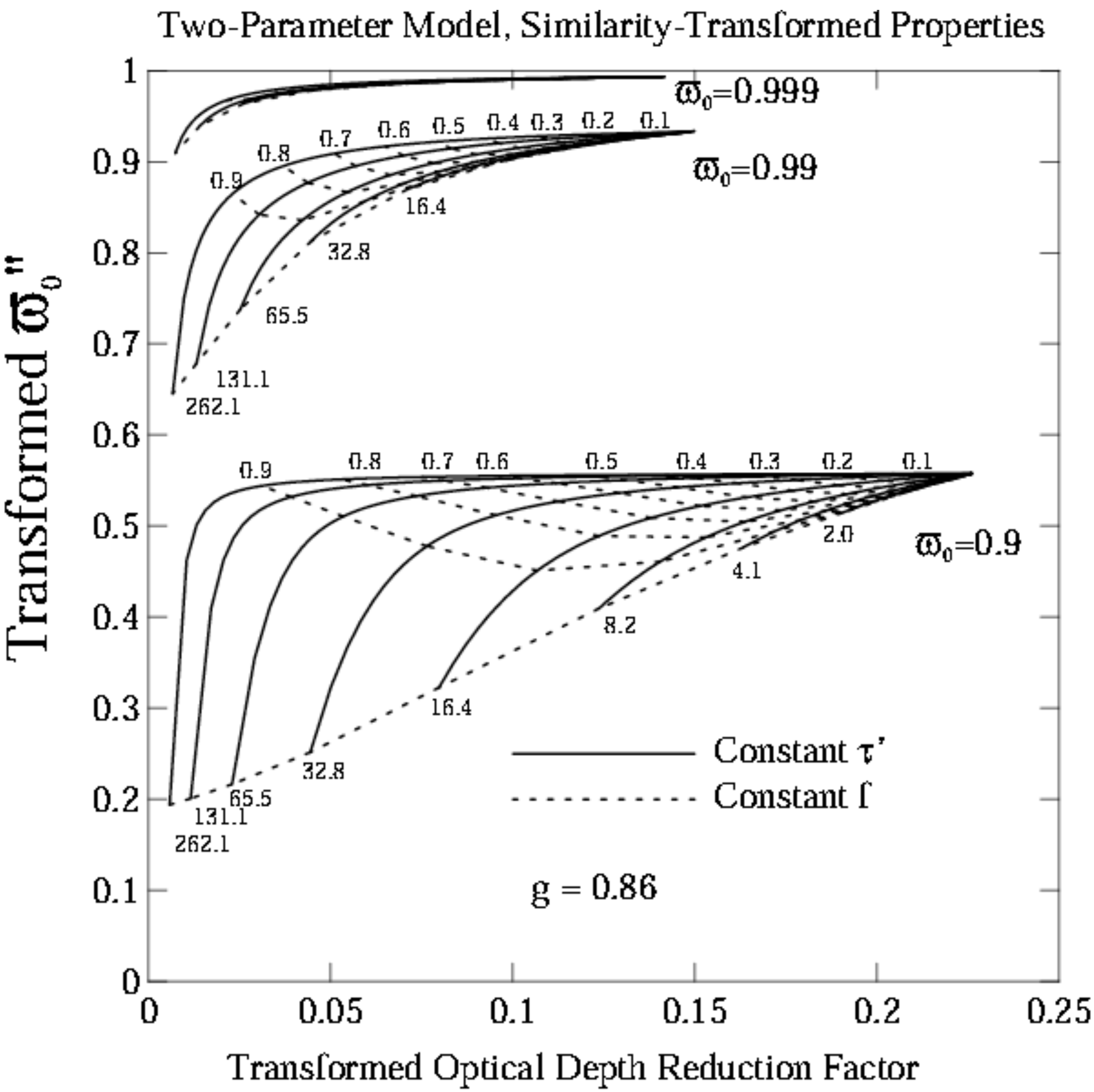
$$\tau^*_{\text{eff}} = \tau_{\text{hom}} + \tau_{\text{inhom}} \equiv \phi''\sigma\overline{W} \tag{37}$$

$$\varpi_0'' = \frac{\tau_{\text{hom}}\varpi_0 + \tau_{\text{inhom}}\varpi_0'}{\tau_{\text{hom}} + \tau_{\text{inhom}}} \tag{38}$$

$$g'' = \frac{\tau_{\text{hom}}\varpi_0 g + \tau_{\text{inhom}}\varpi_0' g'}{\tau_{\text{hom}} + \tau_{\text{inhom}}}, \tag{39}$$



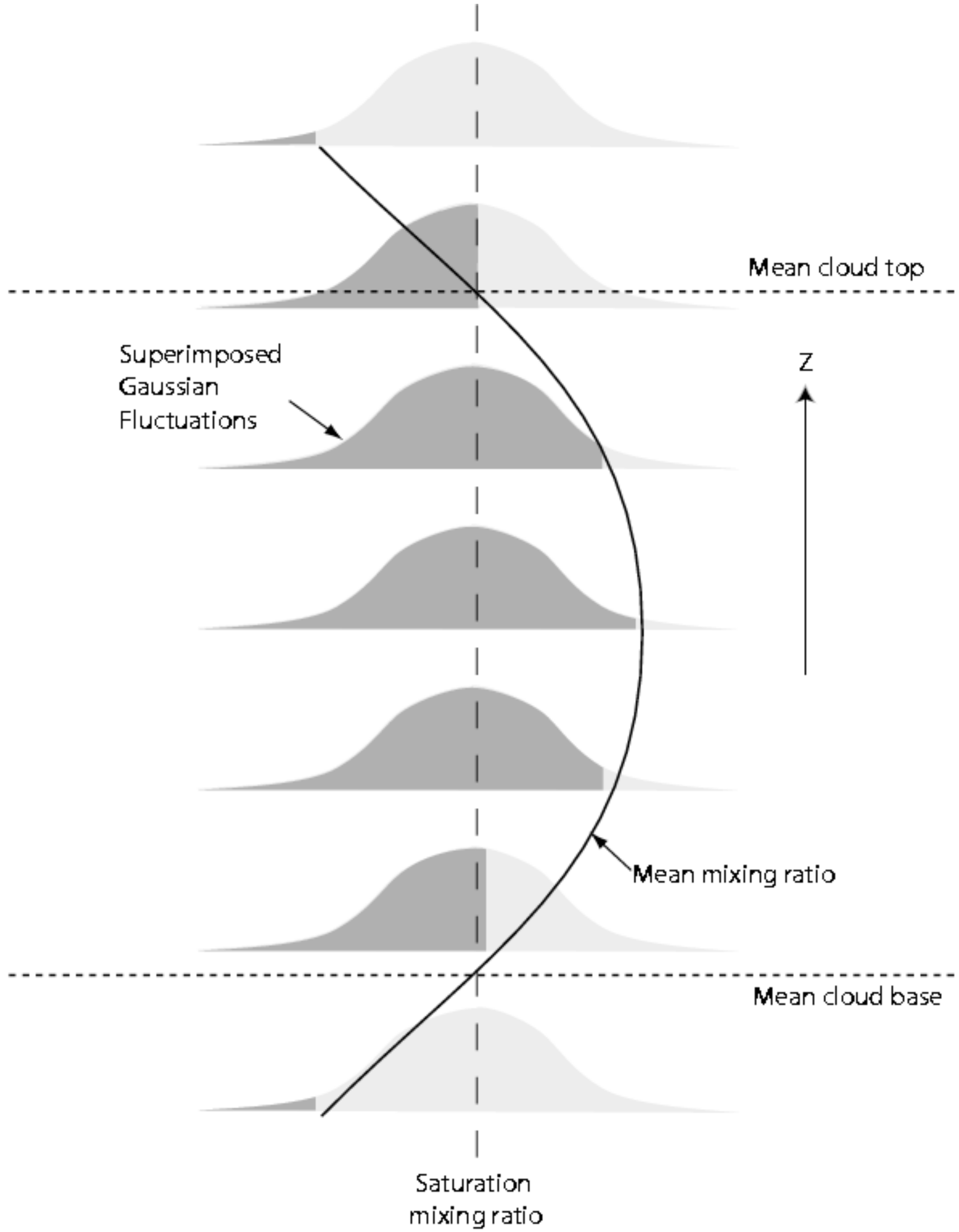
Two-Parameter ISC Model
Renormalized Asymmetry g''
Optical Depth Reduction Factor φ''
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
0
0.2
0.4
0.6
0.8
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.5
1.0
2.0
4.1
8.2
16.4
32.8
65.5
131.1
262.1
Constant τ'
Constant f
ϖ0 = 1.000
g = 0.86

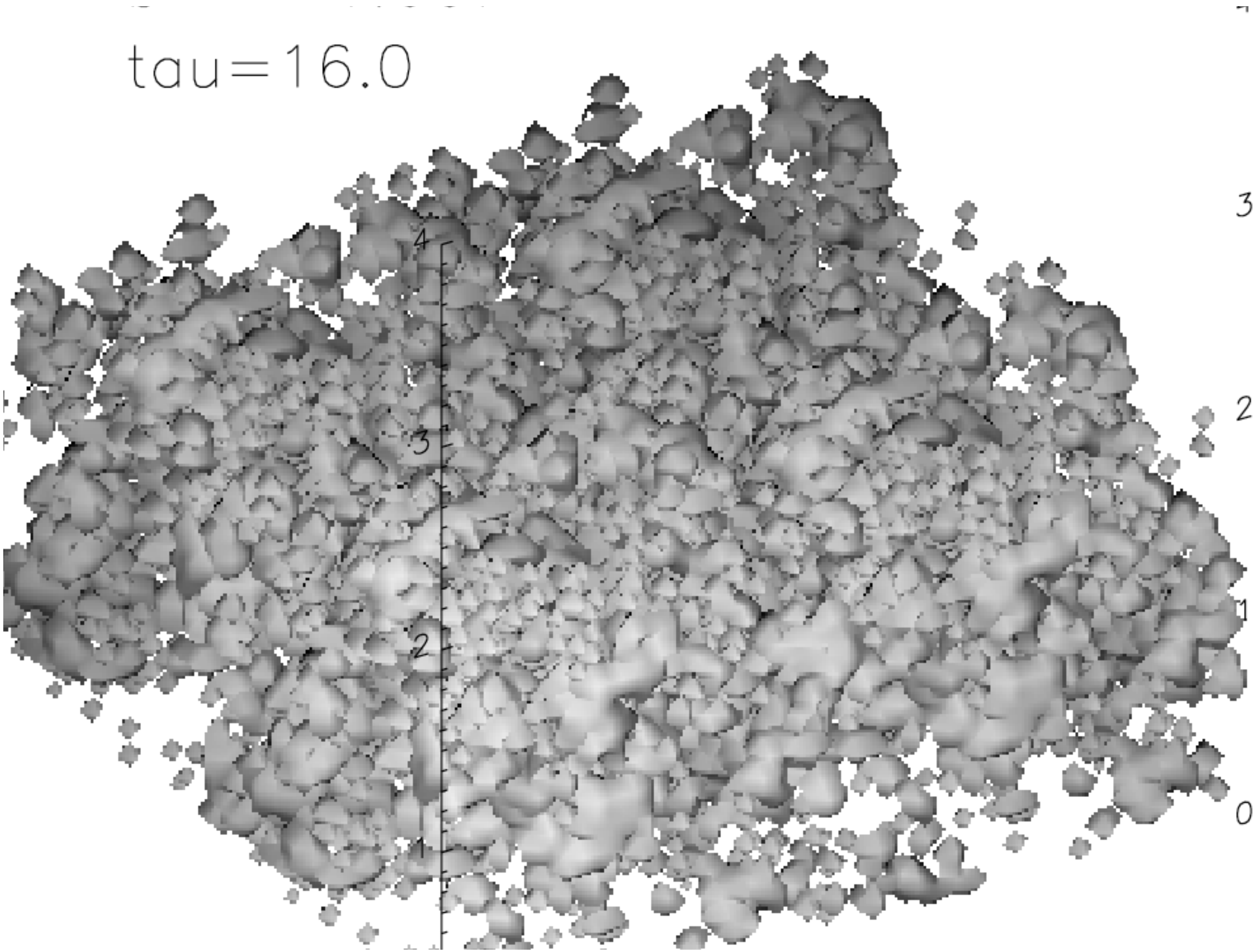


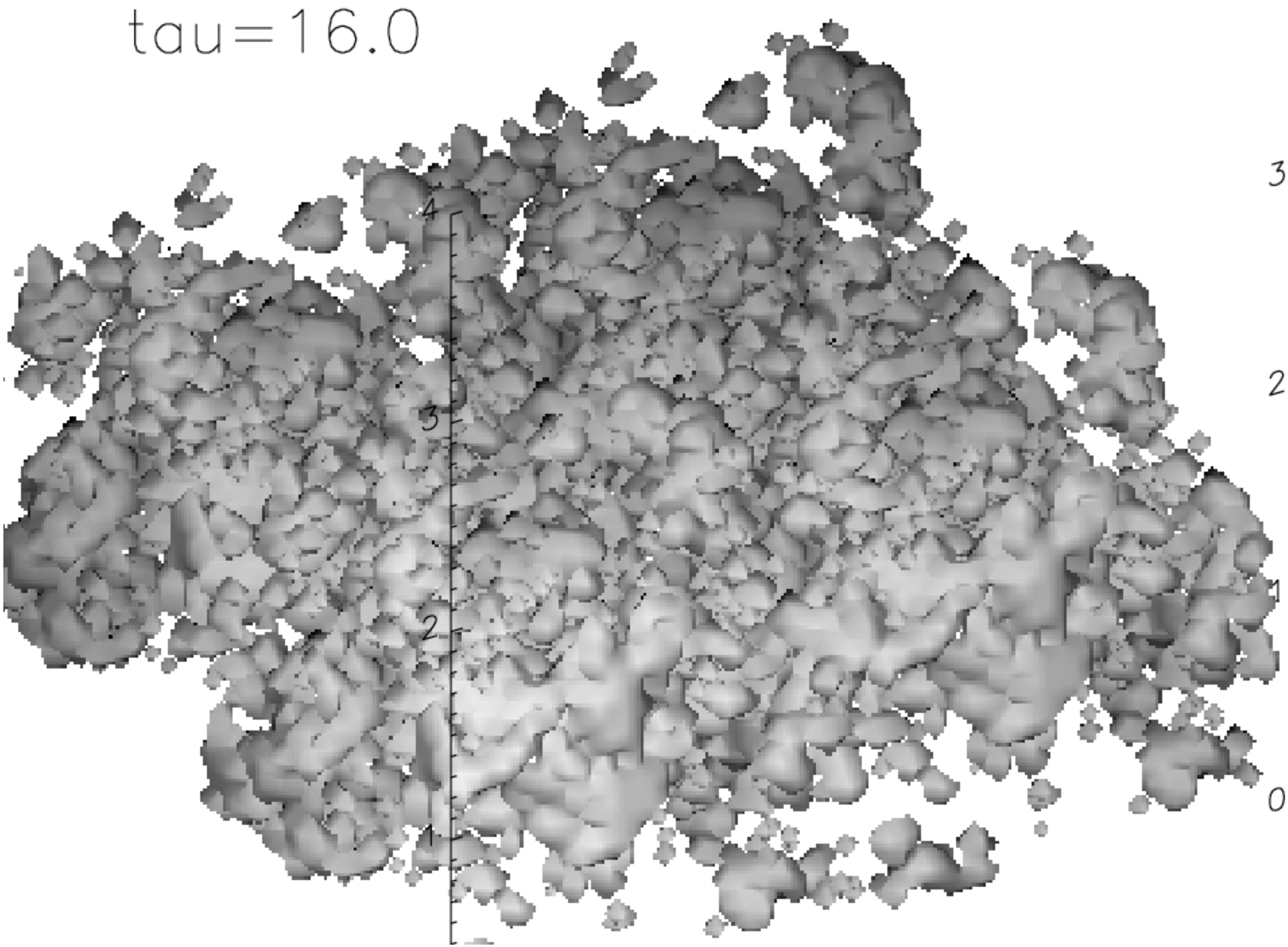
Two-Parameter Model, Similarity-Transformed Properties
Transformed ϖ₀"
Transformed Optical Depth Reduction Factor
ϖ₀=0.999
ϖ₀=0.99
ϖ₀=0.9
Constant τ'
Constant f
g = 0.86
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
16.4
32.8
65.5
131.1
262.1
2.0
4.1
8.2
0
0.05
0.1
0.15
0.2
0.25
1

Next step – can ISC model be shown to have any applicability to "realistic" inhomogeneous cloud structures?

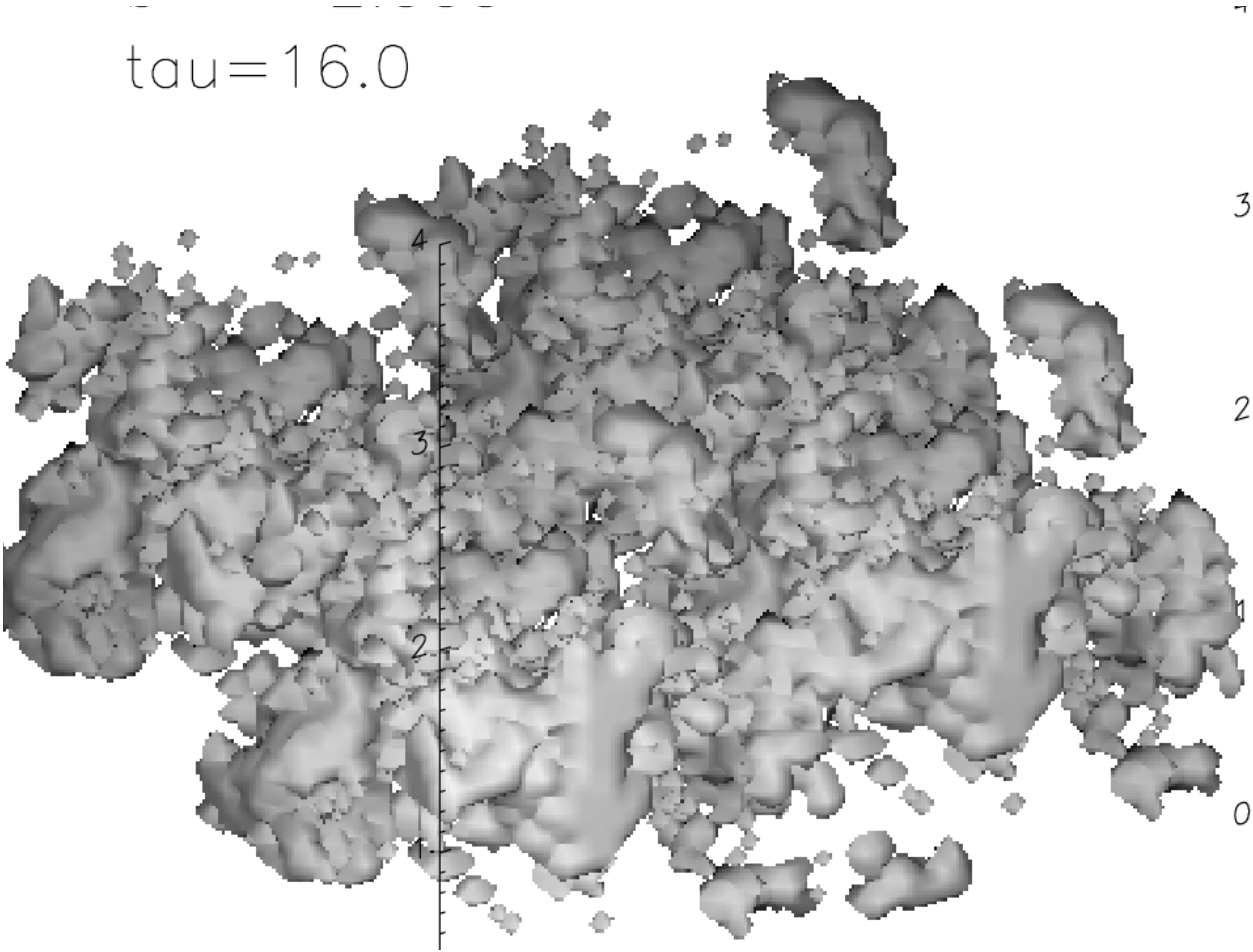
- Generate simulated 3-D structures using filtered white noise
- Perform Monte Carlo calculations of area-averaged fluxes
- *Choose* the two ISC model parameters to match results for non-absorbing case.
- *Test* against variations in single-scatter albedo

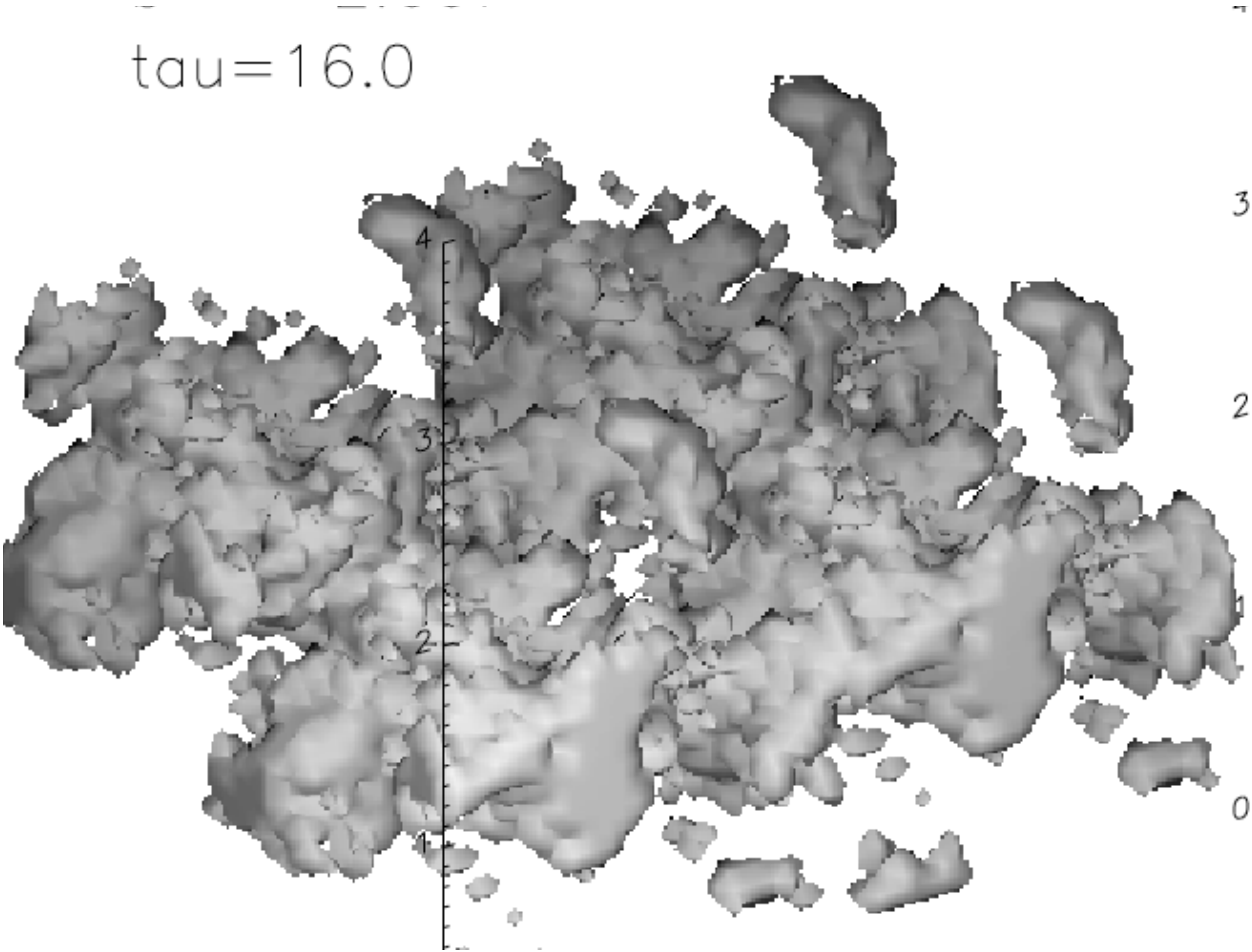

Mean cloud top
Superimposed
Gaussian
Fluctuations
Z
Mean mixing ratio
Mean cloud base
Saturation
mixing ratio


tau=16.0
4
3
2
1
3
2
1
0

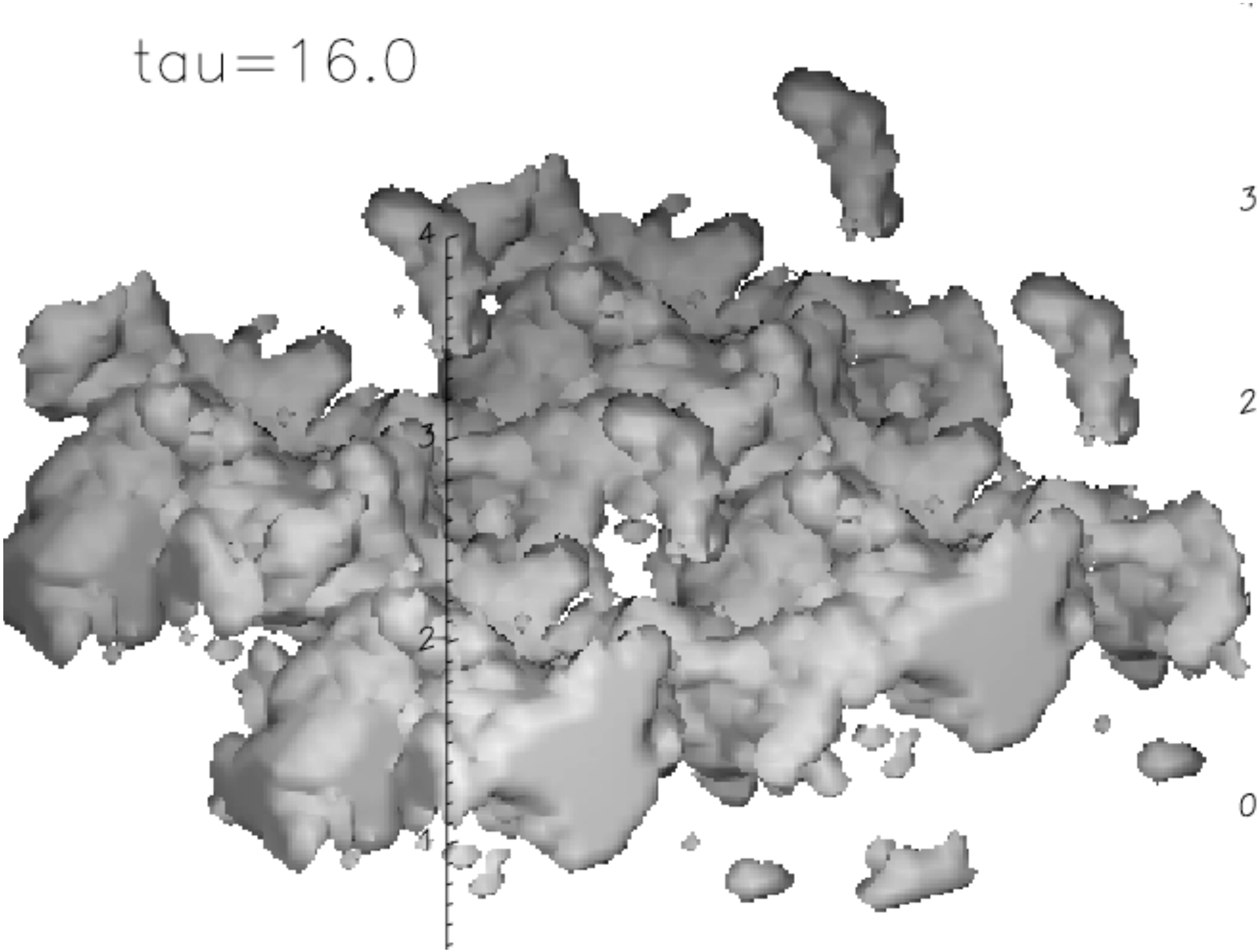


tau=16.0
4
3
2
1
3
2
1
0

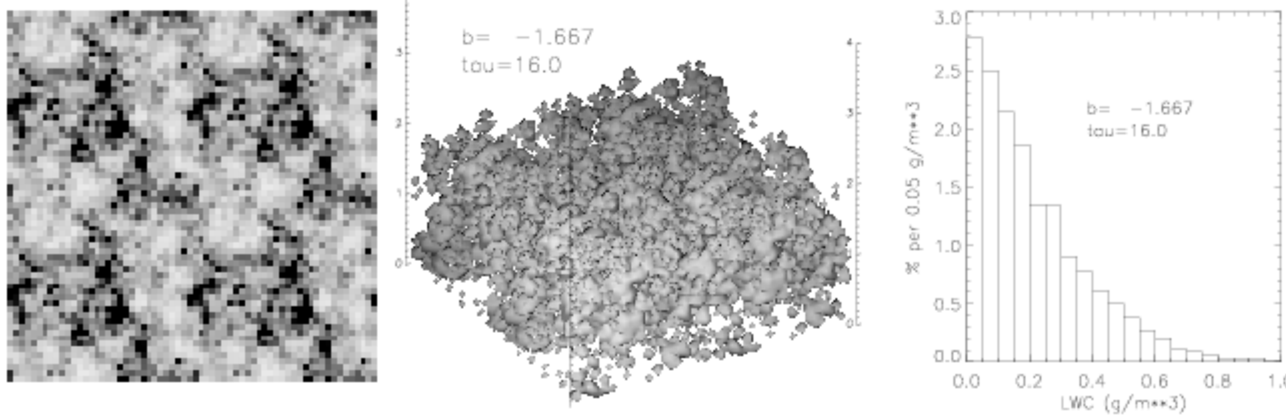

tau=16.0
4
3
2
1
3
2
1
0



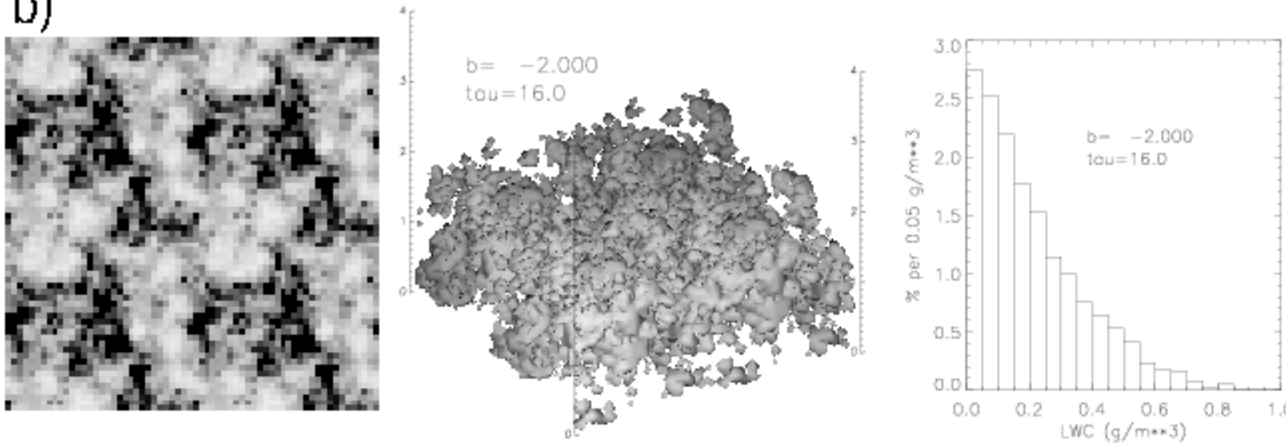
tau=16.0
4
3
2
1
3
2
1
0


tau=16.0
4
3
2
1
3
2
0

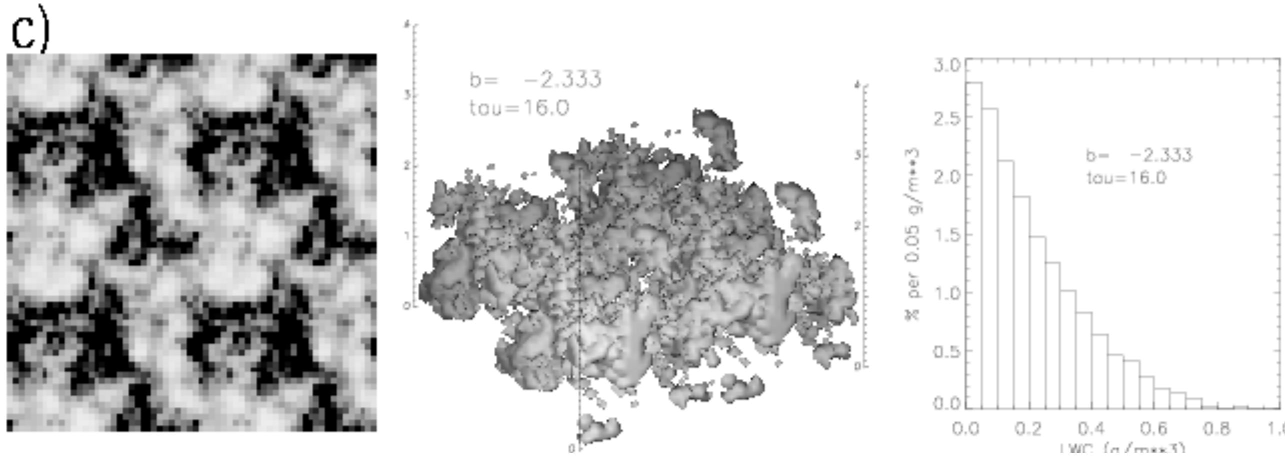
a)


b= −1.667
tau=16.0
b= −1.667
tau=16.0
% per 0.05 g/m**3
LWC (g/m**3)

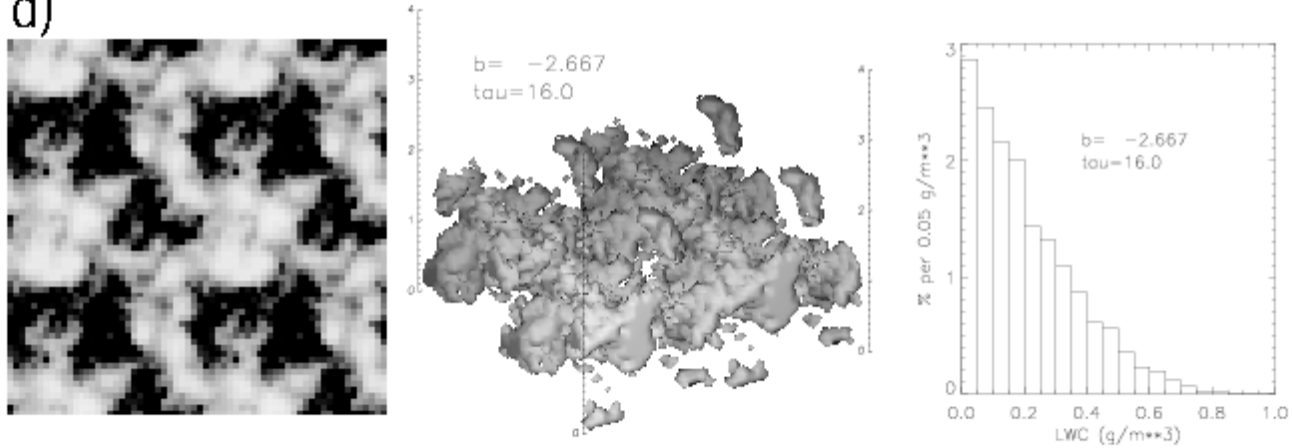
b)


b= −2.000
tau=16.0
b= −2.000
tau=16.0
% per 0.05 g/m**3
LWC (g/m**3)

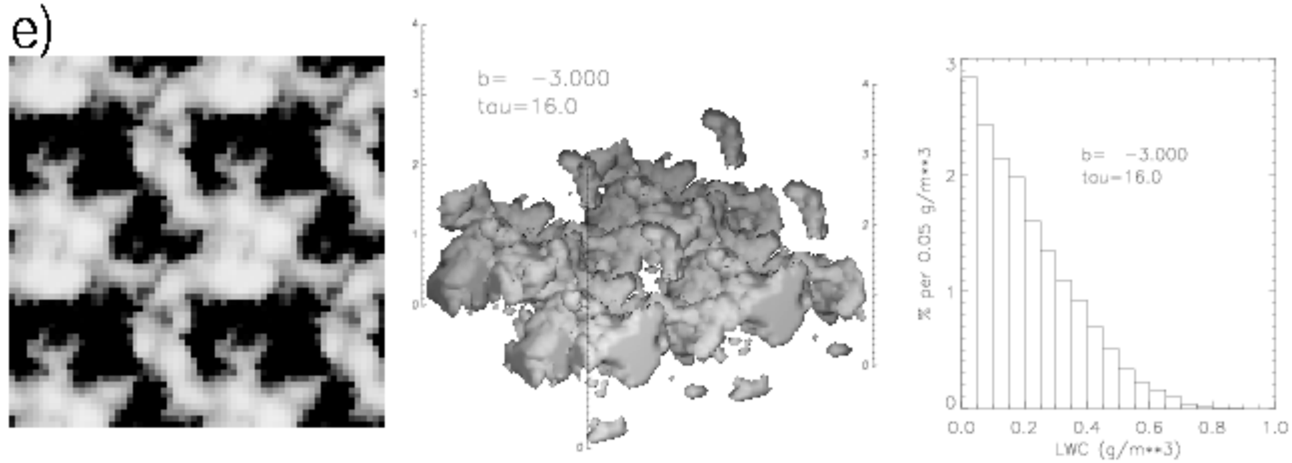
c)


b= −2.333
tau=16.0
b= −2.333
tau=16.0
% per 0.05 g/m**3

d)


b= −2.667
tau=16.0
b= −2.667
tau=16.0
% per 0.05 g/m**3
LWC (g/m**3)

e)


b= −3.000
tau=16.0
b= −3.000
tau=16.0
% per 0.05 g/m**3
LWC (g/m**3)

Case 1: $b = -5/3$, $\overline{\tau}* = 16$, $g = 0.86$, $\mu_0 = 0.5$
Fitted model parameters: $f = 0.935$, $\tau' = 11.81$
Renormalized optical depth: $\tau*_{\mathrm{eff}} = 3.024$

ϖ_0	ϖ_0''	g''		Reflect.	Dir. trans.	Dif. trans.	Tot. trans.	Absorp.
1.000	1.000	0.547	ISC	0.580	0.002	0.418	0.420	0.000
			MC	0.580	0.002	0.418	0.420	0.000
			IPA	0.567	0.067	0.366	0.433	0.000
0.999	0.995	0.550	ISC	**0.561**	0.002	**0.404**	**0.406**	**0.033**
			MC	0.565	0.002	0.403	0.405	0.030
			IPA	0.550	0.067	0.355	0.422	0.028
0.99	0.950	0.560	ISC	**0.439**	0.002	**0.302**	**0.304**	**0.257**
			MC	0.433	0.002	0.302	0.304	0.253
			IPA	0.454	0.067	0.283	0.350	0.196
0.9	0.652	0.641	ISC	**0.118**	0.002	**0.071**	**0.073**	**0.808**
			MC	0.130	0.002	0.074	0.076	0.794
			IPA	0.178	0.067	0.105	0.172	0.650

Case 2: $b = -2$, $\overline{\tau}* = 16$, $g = 0.86$, $\mu_0 = 0.5$,
Fitted model parameters: $f = 0.959$, $\tau' = 14.80$
Renormalized optical depth: $\tau*_{\mathrm{eff}} = 2.255$

ϖ_0	ϖ_0''	g''		Reflect.	Dir. trans.	Dif. trans.	Tot. trans.	Absorp.
1.000	1.000	0.459	ISC	0.553	0.011	0.436	0.447	0.000
			MC	0.553	0.011	0.436	0.447	0.000
			IPA	0.530	0.112	0.358	0.470	0.000
0.999	0.993	0.462	ISC	**0.535**	0.011	**0.421**	**0.432**	**0.033**
			MC	0.537	0.011	0.421	0.432	0.031
			IPA	0.514	0.112	0.346	0.458	0.028
0.99	0.934	0.475	ISC	**0.412**	0.011	**0.317**	**0.328**	**0.260**
			MC	0.415	0.011	0.320	0.331	0.254
			IPA	0.423	0.012	0.281	0.393	0.184
0.9	0.582	0.582	ISC	**0.104**	0.011	**0.085**	**0.096**	**0.800**
			MC	0.119	0.011	0.097	0.108	0.773
			IPA	0.167	0.112	0.118	0.230	0.603

Case 3: $b = -7/3$, $\overline{\tau}* = 16$, $g = 0.86$, $\mu_0 = 0.5$
Fitted model parameters: $f = 0.977$, $\tau' = 17.40$
Renormalized optical depth: $\tau*_{\mathrm{eff}} = 1.715$

ϖ_0	ϖ''	g''		Reflect.	Dir. trans.	Dif. trans.	Tot. trans.	Absorp.
1.000	1.000	0.359	ISC	0.529	0.032	0.439	0.471	0.000
			MC	0.529	0.032	0.439	0.471	0.000
			IPA	0.491	0.181	0.328	0.509	0.000
0.999	0.991	0.361	ISC	**0.511**	0.032	**0.424**	**0.456**	**0.033**
			MC	0.511	0.032	0.425	0.457	0.032
			IPA	0.475	0.180	0.318	0.498	0.027
0.99	0.914	0.376	ISC	**0.388**	0.032	**0.319**	**0.351**	**0.261**
			MC	0.391	0.032	0.325	0.357	0.252
			IPA	0.388	0.180	0.258	0.438	0.174
0.9	0.504	0.500	ISC	**0.093**	0.032	**0.090**	**0.122**	**0.785**
			MC	0.110	0.032	0.112	0.144	0.746
			IPA	0.153	0.180	0.115	0.295	0.552

Case 4: $b = -8/3$, $\overline{\tau}* = 16$, $g = 0.86$, $\mu_0 = 0.5$
Fitted model parameters: $f = 0.997$, $\tau' = 19.72$
Renormalized optical depth: $\tau*_{\text{eff}} = 1.306$.

ϖ_0	ϖ_0''	g''		Reflect.	Dir. trans.	Dif. trans.	Tot. trans.	Absorp.
1.000	1.000	0.227	ISC	0.505	0.073	0.422	0.495	0.000
			MC	0.505	0.073	0.422	0.495	0.000
			IPA	0.455	0.257	0.288	0.545	0.000
0.999	0.988	0.288	ISC	**0.486**	0.073	**0.407**	**0.480**	**0.034**
			MC	0.488	0.073	0.407	0.480	0.032
			IPA	0.438	0.257	0.278	0.535	0.027
0.99	0.889	0.239	ISC	**0.365**	0.073	**0.303**	**0.376**	**0.259**
			MC	0.368	0.073	0.311	0.384	0.248
			IPA	0.353	0.258	0.224	0.482	0.165
0.9	0.404	0.271	ISC	**0.084**	0.073	**0.081**	**0.154**	**0.761**
			MC	0.104	0.073	0.114	0.187	0.709
			IPA	0.138	0.257	0.103	0.360	0.502

Case 5: $b = -3$, $\bar{\tau}* = 16$, $g = 0.86$, $\mu_0 = 0.5$
Fitted model parameters: $f = 1.0000$, $\tau' = 21.41$
Renormalized optical depth: $\tau*_{\mathrm{eff}} = 1.061$

ϖ_0	ϖ_0''	g''		Reflect.	Dir. trans.	Dif. trans.	Tot. trans.	Absorp.
1.000	1.000	0.154	ISC	0.488	0.120	0.392	0.512	0.000
			MC	0.488	0.120	0.392	0.512	0.000
			IPA	0.428	0.321	0.251	0.572	0.000
0.999	0.985	0.155	ISC	**0.470**	0.120	**0.376**	**0.496**	**0.034**
			MC	0.470	0.120	0.377	0.497	0.033
			IPA	0.412	0.321	0.241	0.562	0.026
0.99	0.865	0.161	ISC	**0.349**	0.120	**0.273**	**0.393**	**0.258**
			MC	0.353	0.120	0.285	0.405	0.242
			IPA	0.331	0.321	0.190	0.511	0.158
0.9	0.309	0.227	ISC	**0.078**	0.120	**0.059**	**0.179**	**0.743**
			MC	0.101	0.120	0.104	0.224	0.675
			IPA	0.127	0.321	0.085	0.406	0.467

Summary

- Conceptual and computational framework validated against direct Monte Carlo simulations.
- Two structural parameters appear sufficient to determine effective (equivalent homogeneous) radiative properties of a fairly broad class of 3-D randomly inhomogeneous cloud volumes.
- Quantitative predictions are (mostly) compatible with those of Cairns et al. despite completely different (and less restrictive!) assumptions

Summary (cont.)

- Applicable as well to 2-D sheets of scattered 3-D elements (optically thin limit in area-average sense)
- Possible applicability to bidirectional reflectance from inhomogeneous layers (through modified phase function).
- Only *optical dimensions* of inhomogeneities matter, NOT pointwise density, NOT geometric dimensions, NOT *details* of geometric structure
  - -> no microscale inhomogeneity effects.

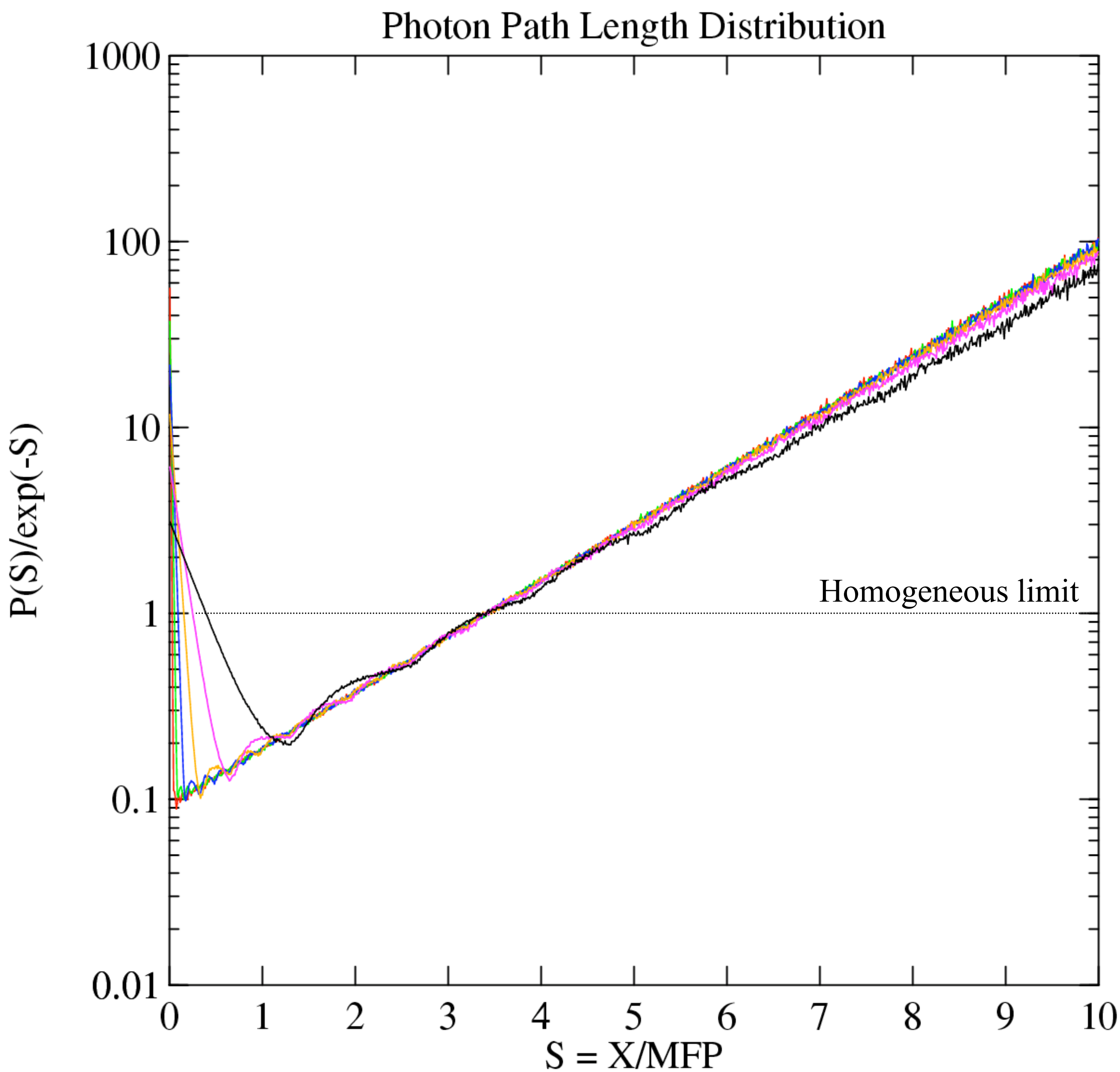
Ongoing/future work

- Explore mapping between model parameters and
  - measurable cloud properties.
  - photon path length distributions
- Empirical determination of “typical” model parameters for actual clouds
  - Matching of actual and predicted fluxes
- Explore transition between 2-D (IPA) and 3-D (ISC) inhomogeneous structures – hybrid?

Photon path length distributions Experiment #1

- Isolated cloudlets of tau-box=4 occupying variable fractions of 50x50x50 domain
- ISC model predicts (MC confirms) **no dependence** on fraction << 1

  1: 1%
  2: 2%
  3: 4%
  4: 8%
  5: 16%
  6: 32%


Photon Path Length Distribution
1000
100
10
1
0.1
0.01
P(S)/exp(-S)
Homogeneous limit
0
1
2
3
4
5
6
7
8
9
10
S = X/MFP

Experiment #2

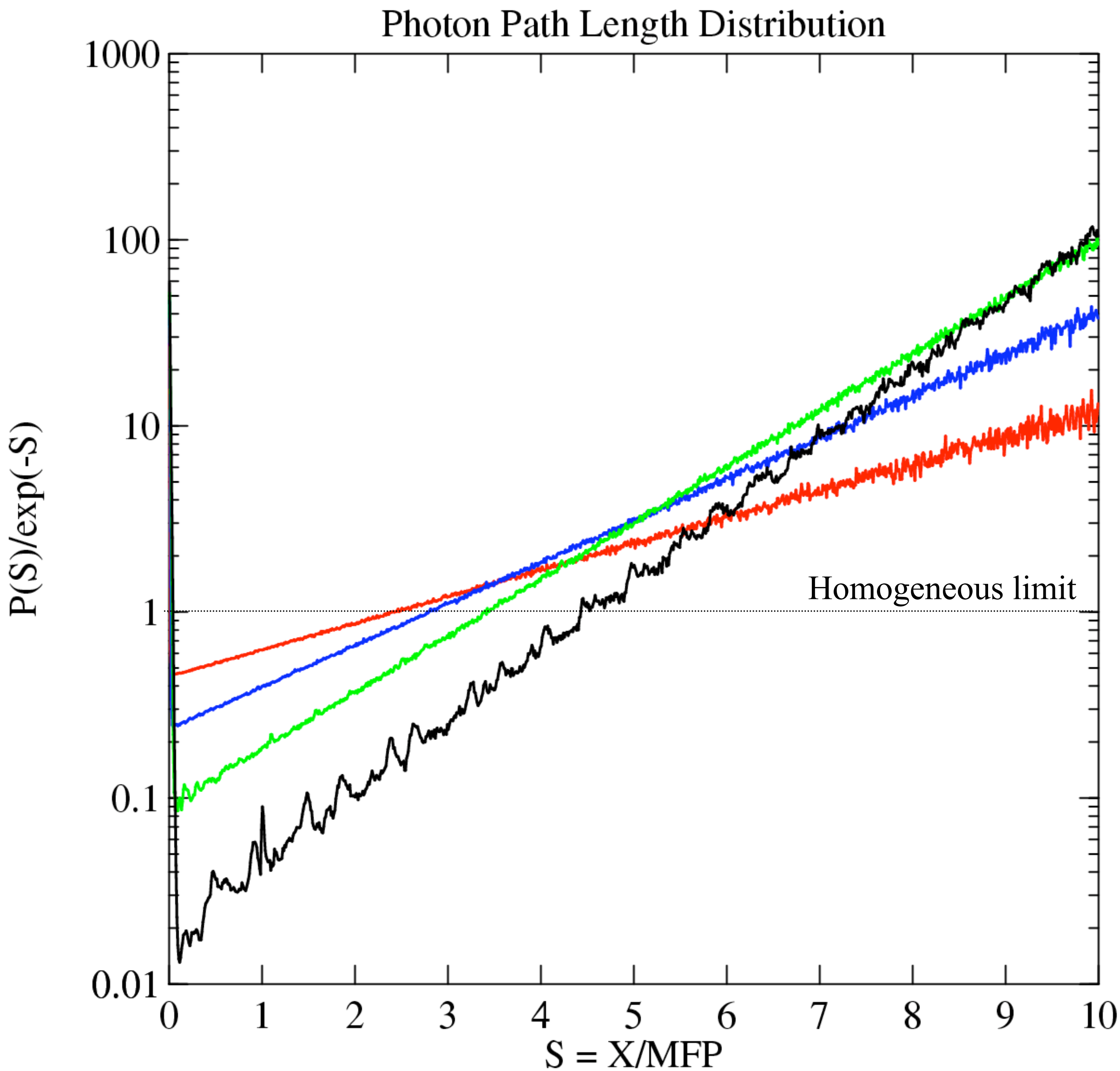
Isolated cloudlets occupying 1% of domain (mimics ISC geometric assumptions)

1: tau-box = 1

2: tau-box = 2

3: tau-box = 4

4: tau-box = 10



Photon Path Length Distribution
1000
100
10
1
0.1
0.01
P(S)/exp(-S)
Homogeneous limit
0
1
2
3
4
5
6
7
8
9
10
S = X/MFP